

## FLOW OF SUSPENSIONS WITH A HIGHLY DISPERSE STRUCTURE-FORMING FRACTION

A. A. Potanin, V. E. Chernomaz, V. M. Tarakanov, and N. B. Ur'ev

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*A relation is obtained to describe the effect of the volume content of particles on the effective viscosity of a concentrated monodisperse suspension of spherical particles in a medium which can be regarded as a power-law fluid. It is shown that this effect becomes weaker as the nonlinearity of the medium increases. The analysis is performed on the basis of the proposition that the deciding role is played by the lubricant action of thin interlayers of the medium between the surfaces of the particles. These results are used to calculate the viscosity of a bidisperse suspension containing a highly dispersed colloidal fraction and a coarsely dispersed fraction (filler). The viscosity is calculated as a function of the ratio of the two fractions. The calculated results are compared with experimental data for model systems.*

**Introduction.** Disperse systems that can be regarded as suspensions of particles in a non-Newtonian fluid are used in many production processes. An example of such a system is a filled polymer [1]. Also, most such suspensions are polydisperse. Here, particles of the fine fractions can be regarded as a certain homogeneous medium into which particles of coarse fractions have been placed. Since a suspension of highly dispersed particles is structured and therefore non-Newtonian, it can be concluded that the model of a suspension of particles in a non-Newtonian medium is the most realistic basis for analyzing the more complex rheological properties of suspensions.

The most important problem in the rheology of disperse systems is developing scientific principles for optimizing the granulometric composition of polydisperse systems. The solution of this problem is important for the development of many engineering processes. For example, the development of means of transporting highly concentrated water-coal suspensions has shown that the highest degree of fluidity is attained with two-stage crushing. In this case, the suspension contains two fractions — a highly dispersed fraction ( $\delta_1 < 10 \mu\text{m}$ ) and a coarsely dispersed fraction ( $250 \mu\text{m} > \delta_2 > 10 \mu\text{m}$ ) [2]. Similar results have been obtained for suspensions of coal in methanol [3]. One feature of the suspensions used in these and many other engineering processes is that they contain two fractions which differ significantly in terms of size. Here, the highly dispersed fraction contains particles of colloidal dimensions, which results in the formation of a structure and the manifestation of non-Newtonian properties [4]. The particles of the coarsely dispersed fraction, conversely, are large enough so that all forms of interaction between them that are not hydrodynamic can be ignored. Since  $\delta_1 \ll \delta_2$ , it can be assumed that the particles of the coarse fraction have been placed in a continuous non-Newtonian medium. The role of this medium is played by the highly dispersed fraction.

**Suspension of Spheres in a Generalized Newtonian Medium.** We will determine the viscosity of a suspension of spherical particles in a medium which, for the sake of definiteness, we will assume to be a power-law fluid, i.e., we will assume that its viscosity depends on the strain rate in accordance with the law

$$\eta = \frac{K}{S^n}, \quad (1)$$

where  $S \equiv (2\mathbf{D}:\mathbf{D})^{\frac{1}{2}}$ ,  $\mathbf{D} = \frac{1}{2}((\nabla\mathbf{v}) + (\nabla\mathbf{v})^T)$  is the strain-rate tensor. In the pure shear flow usually obtained in a viscosimeter,  $S = \dot{\epsilon}$  is the strain rate and  $\epsilon$  is the strain. With the appropriate choice of values for the parameters  $K$  and  $n$ , Eq. (1) reliably describes a broad range of polymer solutions and highly dispersed suspensions.

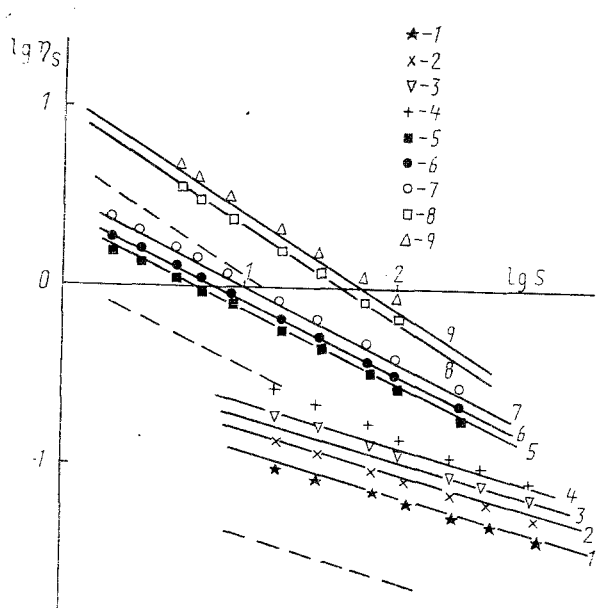


Fig. 1. Dependence of the viscosity of a coarsely dispersed suspension of PMMA on shear rate and concentration in different media. The solid lines denote calculated results, while the points show experimental data and the dashed lines show the experimental relations for the corresponding media. The following was used in these media: 7% suspension of bentonite ( $n = 0.3$ ):  $\varphi = 35\%$  (1); 40 (2); 45 (3); 47 (4); 10% suspension of bentonite ( $n = 0.5$ ):  $\varphi = 25\%$  (5); 30 (6); 40 (7); solution of SAKAP ( $n = 0.6$ ):  $\varphi = 30\%$  (8); 50 (9).  $\log \eta_s$ , Pa-sec;  $\log S$ ,  $\text{sec}^{-1}$ .

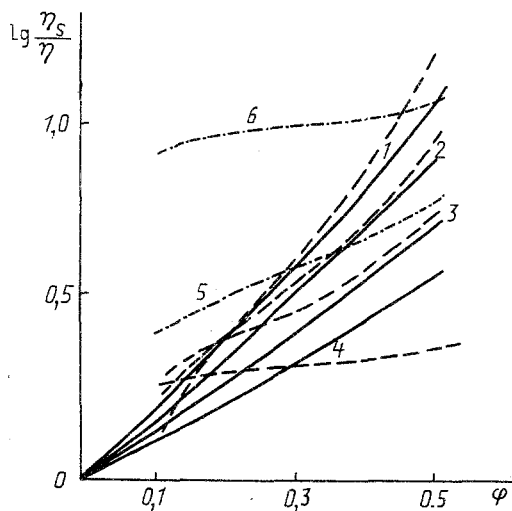


Fig. 2. Results of calculations performed with the corrected lubrication model (dashed curves) and the free-surface model (solid curves) at  $n = 0$  (1); 0.2 (2); 0.4 (3); 0.6 (4); the dot-dash curves 5, 6 correspond to the uncorrected lubrication model with  $n = 0.4$  and 0.6.

A fairly rigorous analysis of the rheological properties of suspensions can be made only for volume contents of particles  $\varphi \ll 1$  and only then for Newtonian media. At high  $\varphi$ , the viscosity of the suspension can be calculated only on the basis of model representations on the character of the relative motion of the particles. At  $\varphi \rightarrow \varphi_m \approx 0.6$  (the value of  $\varphi$  corresponding to a dense random packing), when the mean distance between the surfaces of particles  $h$  is small:

$$h \equiv ((\varphi_m/\varphi)^{1/3} - 1)^{-1} \ll a, \quad (2)$$

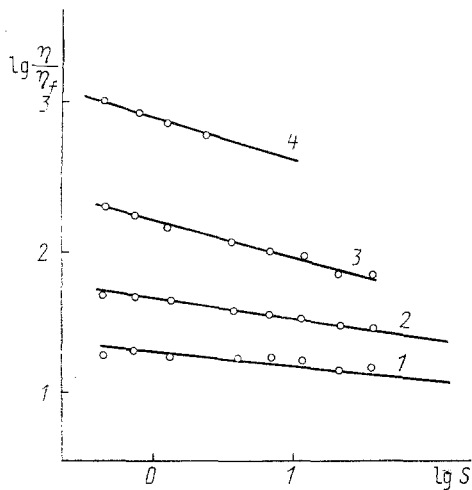


Fig. 3

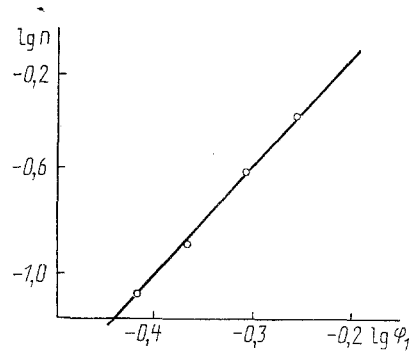


Fig. 4

Fig. 3. Rheological curves for a marshalite-glycerin suspension; the points show experimental data, while the lines show data obtained from a linear regression analysis: 1)  $\varphi_1 = 37\%$ ; 2) 42; 3) 47; 4) 52%.

Fig. 4. Dependence of the exponent  $n$  on  $\varphi_1$ ; the points correspond to the experiments in Fig. 1, while the line shows the result of linear regression analysis.

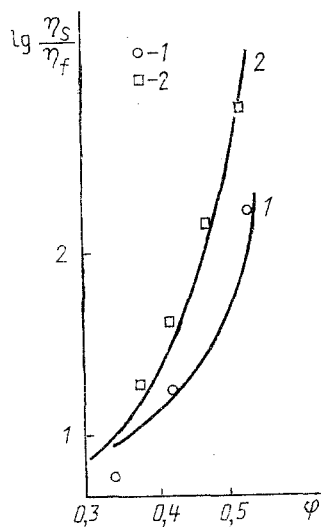


Fig. 5

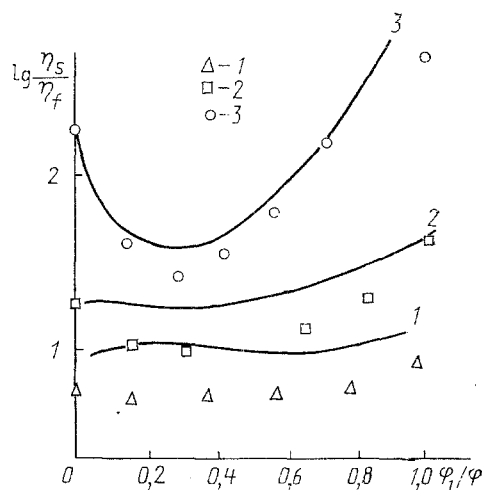


Fig. 6

Fig. 5. Viscosity relation of glass-bead-glycerin (1) and marshalite-glycerin (2) systems at  $S = 2.2 \text{ sec}^{-1}$ ; the points show experimental data, while the curves show results calculated from Eq. (24).

Fig. 6. Dependence of the viscosity of a bidisperse suspension on the relative content of the fine fraction with  $\varphi = 0.33$  (1); 0.42 (2); 0.52 (3); the points show experimental results, while the curves show theoretical results.

the effect of the fluid on the motion of the particles can be reduced to the lubricant effect of its thin interlayers between particle surfaces [5, 6], i.e., it can be reduced to effective forces  $F$  of hydrodynamic interaction among the particles. If the medium is Newtonian, then  $F$  is determined from the formula [7]:

$$F = \frac{3\pi}{2} \eta a V \left( \frac{a}{h} + 0.9 \ln \frac{a}{h} + \dots \right), \quad (3)$$

Meanwhile, for rheological calculations with  $h/a \ll 1$  in this formula, it is sufficient to retain only the first term of the expansion in  $h/a$ . An analysis made in [5] on the basis of study of a dilatational flow in a certain cellular model led to the following estimate of suspension viscosity for  $\varphi \rightarrow \varphi_m$ :

$$\frac{\eta_s}{\eta} = \frac{3}{8\pi} \frac{FV}{\eta S^2 a^3} \quad (4)$$

Inserting (3) into (4), we obtain the following for Newtonian media

$$\frac{\eta_s}{\eta} = \frac{1,1}{(\varphi_m/\varphi)^{1/3} - 1} \approx \frac{1,1}{1 - (\varphi/\varphi_m)^{1/3}} \quad (5)$$

A similar analysis was made in [8], where it was noted that, within the framework of the given method, Eqs. (4) and (5) can be accurate only to within the numerical coefficient. The latter can be determined on the basis of a comparison with experimental data. Such a comparison [5] indicates that the coefficients used above should be used. It should be noted that attempts made in [6] to refine Eq. (5) on the basis of allowance for the subsequent terms of the expansion in  $h/a$  go beyond the scope of lubrication theory and are incorrect, since the logarithmic term in (3) is dropped in this case.

An important advantage of the given approach is the possibility of generalizing it to the case of suspensions with a non-Newtonian medium of type (1). In this case, concentrated suspensions turn out to be simpler to analyze than dilute suspensions. Performing such a generalization for dilute suspensions is extremely complex. In the present case, it is sufficient that we obtain the relation  $F(V)$  and substitute it into (4). To do this, we will examine the flow of the medium between particle surfaces. Here, we introduce a cylindrical coordinate system  $z, r, \psi$  with its origin on the surface of one of the particles and the  $z$  axis directed along the line of centers. Considering that  $\partial v_r/\partial z \gg \partial v_z/\partial r$ ,  $r \ll (ah)^{1/2}$  (it is this region that makes the main contribution to the integral determining  $F$ ), we have  $S \approx \partial v_r/\partial z$ . This makes it possible to write the equation of motion of the medium in the gap between particles in a manner similar to that used in the solution of the familiar Reynolds problem on the convergence of plane disks in a viscous medium [9]:

$$\frac{\partial}{\partial z} \left[ \eta \left( \frac{\partial v_r}{\partial z} \right) \frac{\partial v_r}{\partial z} \right] = \frac{\partial p}{\partial r} \quad (6a)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{\partial v_z}{\partial z} = 0, \quad (6b)$$

$$\frac{\partial p}{\partial z} = 0. \quad (6c)$$

The boundary conditions have the form

$$v_z = v_r = 0, \quad z = 0; \quad \frac{\partial v_r}{\partial z} = 0, \quad z = \frac{H}{2}; \quad (7)$$

$$v_z = V, \quad z = H; \quad p = 0, \quad r \rightarrow \infty,$$

where  $H = h + r^2/a$  is the local value of the distance between the surfaces of the particles. Here, we will make use of the standard method employed in [10] to change over from the problem of calculating the drag of disks to the corresponding problem for spherical surfaces. The method is based on the fact that the dependence of  $H$  on  $r$  can be ignored when solving the hydrodynamic equations, since  $\partial H/\partial r = 2r/a \ll 1$ . This was already done when we wrote  $S$  in the form  $\partial v_r/\partial z$ , i.e., the flow of the fluid is for the most part radial. Solving (6)-(7), we determine  $p(r)$ . We then calculate  $F$  by integrating  $p$  over the surface of the particles:

$$F = \int_0^\infty p(r) \pi dr^2. \quad (8)$$

Such an approach can be used to obtain an exact expression for the singular term in the expansions of  $F$  in  $h/a$ . Before presenting this solution, we will show how the simplest estimates can be used to obtain an appropriate formula which is accurate to within the numerical coefficient. We note that the given problem contains two parameters with the

dimensions of length: the width of the gap  $h$  (the dimension along the  $z$  axis) and the effective radius of the interaction zone  $(ah)^{\frac{1}{2}}$  (the dimension along the  $r$  axis). Thus, in calculation accurate to within the numerical coefficient, we can make the substitutions

$$\frac{\partial}{\partial z} \rightarrow \frac{1}{h}, \quad \frac{\partial}{\partial r} \rightarrow \frac{1}{\sqrt{ah}}, \quad \int \dots dr^2 \rightarrow ah. \quad (9)$$

Proceeding on the basis of (6a), we then obtain

$$p \sim \eta(S) v_r/h, \quad (10)$$

where  $S \sim v_r/h$ . With allowance for (9), we find from (6b) that

$$v_r \sim V \sqrt{a/h}. \quad (11)$$

Inserting (11) into (10), we will have

$$p \sim \frac{K}{(v_r/h)^n} V \frac{a}{h^2}, \quad (12)$$

From (8) we finally obtain

$$F \sim pah \sim K (V/a)^{1-n} a^2 (a/h)^{1-\frac{3}{2}n}. \quad (13)$$

These calculations are valid for  $n < 2/3$ . In the opposite case, the integral in (8) diverges. It is easy to understand the physical significance of this divergence. The contribution of the region  $r \ll (ah)^{\frac{1}{2}}$  to  $F$  ceases to be decisive as the nonlinearity of the medium increases, since although  $h$  is small in this region,  $\eta$  is also small due to the high values of  $S$ . At  $n \geq 2/3$ , the resistance offered to the relative motion of the particles is determined not by the lubricant effect of the thin interlayers (these regions are exceptional, since the viscosity of the medium in them is low), but by the motion of the fluid relative to the surface of the particles as a whole, i.e., it is as if the fluid flowed over the surface of each particle individually in this case. Here, the role of hydrodynamic interaction is small. Inserting (13) into (4), we find that

$$\frac{\eta_s}{\eta} = C(n) \left( \frac{2a}{h} \right)^{1-\frac{3n}{2}} = \frac{C(n)}{((\varphi_m/\varphi)^{1/3} - 1)^{1-\frac{3}{2}n}} \approx \frac{C(n)}{(1 - (\varphi/\varphi_m)^{1/3})^{1-\frac{3}{2}n}}, \quad (14)$$

where the coefficient  $C(n)$  can be determined on the basis of the exact solution of (6)-(7). This solution yields

$$p(r) = \left[ \frac{(3-2n)(2-n)}{1-n} \right]^{1-n} K \left( \frac{V}{a} \right)^{1-n} \int_{r/a}^{\infty} \frac{\xi^{1-n} d\xi}{\left( \xi^2 + \frac{h}{a} \right)^{3-2n}}. \quad (15)$$

A similar expression was obtained in [11]. However, an error that was made in the calculation of (8) caused the exponent in (14) to take the value  $1 - 2n$ . Correcting this error and performing calculations with the use of the definition of the beta function  $B(\dots, \dots)$  [12], we obtain

$$F = \frac{\pi}{2} \left[ \frac{(3-2n)(2-n)}{1-n} \right]^{1-n} B \left( 2 - \frac{n}{2}, 1 - \frac{3n}{2} \right) \times K \left( \frac{V}{a} \right)^{1-n} a^2 \left( \frac{a}{h} \right)^{1-\frac{3n}{2}}. \quad (16)$$

The viscosity of the suspension is described by Eq. (14). We find the following for the coefficient  $C(n)$  in this equation

$$C(n) = \frac{3}{8} 2^{n/2} \left[ \frac{(3-2n)(2-n)}{1-n} \right]^{1-n} B \left( 2 - \frac{n}{2}, 1 - \frac{3n}{2} \right). \quad (17)$$

We performed experiments with monodisperse suspensions of spherical particles of PMMA ( $2a = 350 \mu\text{m}$ ) in different media that satisfied Eq. (1) (a suspension of sodium bentonite and a solution of a copolymer of polyacrylic acids (SAKAP)). The measurements were made on a "Reotest-2" viscosimeter in a coaxial-cylinder geometry. The experimental data was compared with the results of calculations performed with Eq. (14). The comparison is shown in Fig. 1, from which it is evident that the effective viscosity of the suspension in a generalized Newtonian medium is proportional to the effective viscosity of the medium, i.e., is characterized by the same dependence on shear rate. This circumstance was also noted in [1]. The dependence  $\eta/(\dot{\varphi})$  becomes weaker with an increase in the nonlinearity index  $n$ . Thus, at  $n$  close to  $2/3$  (SAKAP), it is almost indistinguishable. As regards the numerical value of the coefficient  $C(n)$ , it is determined from Eq. (17) only at  $n \leq 0.3$  (for 7% bentonite). At  $n > 0.3$ , the coefficient  $C$  remains approximately constant  $C \approx C(0.3) = 1.77$ . This value was determined from the best agreement with the experimental data.

It should be noted that a somewhat different approach to the determination of the viscosity of the suspension in medium (1) was proposed in [13]. It is based on generalization of the model of a free Happel surface [14]. In this model, the hydrodynamic interaction of a particle with its neighbors is accounted for by introducing special boundary conditions for the surface of the spherical cell surrounding the given particle. Since it is impossible to obtain an exact solution to the equations of motion of the fluid (1) in spherical coordinates, the author of [13] proposed an approximate method in which energy dissipation in the cell and, thus, viscosity are determined with the same velocity field as for the corresponding Newtonian fluid. The results of numerical calculations from [13] are compared with results calculated from Eq. (14) in Fig. 2. It is evident from the figure that the two models yield similar results when nonlinearity is not too great ( $n \leq 0.3$ ). At large  $n$ , lubrication theory gives exaggerated results. The agreement between the models is improved somewhat if we assume that  $C \approx C(0.3)$  at  $n > 0.3$ . The validity of this assumption is illustrated by the experimental data. A deviation occurs only at  $n > 0.5$ . The experiments, meanwhile, show that lubrication theory is preferable in this case.

**Bidisperse Suspensions.** The above analysis was purely hydrodynamic. Now let us examine its application to the solution of a colloid chemistry problem involving the determination of the effective viscosity of a suspension containing coarsely dispersed particles in a suspension of colloidal particles. In the general case, the effective viscosity  $\eta$  of a colloidal system is a function of two dimensionless parameters corresponding to the main microscopic processes that determine the rheology of these systems [15, 16]: the process of particle—particle interaction  $\tau_F S$  and the process of Brownian motion  $\tau_B S$ . Here,  $\tau_F = C_F \eta_\infty \delta_1^2 F_c$  and  $\tau_B = C_B \eta_\infty \delta_1^3 / kT$  are the characteristic times of these processes and  $\eta_\infty$  is the hydrodynamic component of the viscosity of the suspension, i.e., the viscosity at high values of  $S$  — when the structure has been completely destroyed and the suspension is Newtonian. The parameter  $\tau S$  (where  $\tau = \tau_F$  or  $\tau = \tau_B$ ) characterizes the ratio of the characteristic time of a micro-process to the characteristic time of deformation  $1/S$ . Thus, this parameter is the micro-rheological analog of the Deborah number [15]. The viscosity of the colloidal suspension as a whole can often be represented in the form [16]

$$\eta = \eta_\infty + \frac{\eta_\infty}{\frac{\eta}{\eta_0} + (\tau S)^n} \quad (18)$$

For highly concentrated suspensions, there is usually no low-velocity Newtonian limit and  $\eta_0 \rightarrow \infty$  [17]. Thus, at  $\tau S \ll 1$  we can write

$$\frac{\eta}{\eta_\infty} = (\tau S)^{-n}, \quad (19)$$

i.e., the suspension manifests the properties of a power-law fluid,  $K = \eta_\infty \tau^{-n}$ . Of the two processes mentioned above, the dominant process will be that which takes less time. For example, the particle—particle interaction process dominates at  $\tau_F \ll \tau_B$ , i.e.,  $F_b \gg kT$ . Assuming that the interaction takes place as a result of undelayed molecular attraction characterized by  $A \sim 10^{-21}$  J, at  $h_b \ll \delta_1$  we have [18]

$$F_b = \frac{A \delta_1}{24 h_b^2} \quad (20)$$

Then the condition  $\tau_F \ll \tau_B$  is satisfied at  $\delta/h_b \gg (24kT/A)^{1/2}$ . Since the distance  $h_b$  corresponding to the coagulation of particles is usually less than  $0.1 \mu\text{m}$ , the last inequality is always satisfied for particles of  $1 \mu\text{m}$  diameter or larger.

It is such systems that we will concern ourselves with below. In this case, in (19)  $\tau \rightarrow \tau_F$ . The exponent  $n$  in (19) is a function of the volume content of the particles of the fine fraction  $\varphi_1$ . It was found that  $C_F = 6.4$  and  $n = 1/2$  for limitingly concentrated systems ( $\varphi_1 \rightarrow \varphi_m$ ) [19]. It should be expected that  $n$  will decrease as dilution proceeds. Meanwhile,  $n \rightarrow 0$  at  $\varphi_1 \rightarrow 0$ .

The viscosity of a bidisperse suspension as a whole is determined from Eq. (14):

$$\frac{\eta_s}{\eta} = \frac{C(n)}{(1 - (\varphi_2/\varphi_m)^{1/3})^{1 - \frac{3n}{2}}} \quad (21)$$

If the finely dispersed fraction is stabilized, then  $\eta \rightarrow \eta_\infty$  and  $n = 0$ . Meanwhile,  $\eta_\infty$  is determined from a formula of the form (5):

$$\frac{\eta_\infty}{\eta_f} = \frac{1,1}{1 - (\tilde{\varphi}_1/\varphi_m)^{1/3}}, \quad (22)$$

where the local value of the volume content of the finely dispersed fraction in the space between particles of the coarse fraction  $\tilde{\varphi}_1$  is determined from the equation

$$\frac{1}{\tilde{\varphi}_1} = \frac{1}{\varphi_1} - \frac{\varphi_2}{\varphi_1}, \quad (23)$$

where  $\varphi_1 \equiv V_1/V_0$ ,  $\varphi_2 \equiv V_2/V_0$ , while  $V_1$ ,  $V_2$ , and  $V_0$  are the total volumes of particles of the fine and coarse fractions and the total volume of the system. If a suspension of particles of a finely dispersed fraction is non-Newtonian due to structuring, then general equation (21) should be used. The value of  $\eta$  in this equation is determined from Eq. (19) with allowance for (22):

$$\frac{\eta_s}{\eta_f} \approx \frac{C(n(\tilde{\varphi}_1)) \left( \frac{C_F F_c}{\eta_f S \delta_1^2} \right)^{n(\tilde{\varphi}_1)}}{\left[ 1 - \left( \frac{\varphi_2}{\varphi_m} \right)^{1/3} \right]^{1 - \frac{3}{2} n(\tilde{\varphi}_1)} \left[ 1 - \left( \frac{\tilde{\varphi}_1}{\varphi_m} \right)^{1/3} \right]^{1 - n(\tilde{\varphi}_1)}} \quad (24)$$

The above formula completely describes the rheological properties of a bidisperse suspension if the rheological properties of the suspension of the finely dispersed fraction (matrix) are known, i.e., if the relation  $n(\tilde{\varphi}_1)$  and the quantity  $C_F F_b/\delta_1^2$ , corresponding to the strength of aggregates [19], are both known.

To demonstrate the potential uses of Eq. (24), we will examine the results of experiments conducted with a bidisperse suspension consisting of glass beads ( $\delta_2 \approx 500 \mu\text{m}$ ), marshalite ( $\delta_1 \approx 10 \mu\text{m}$ ), and glycerin. The experiments were conducted on a "Reotest-2" viscosimeter in a coaxial-cylinder geometry. The choice of a dispersion medium with a high viscosity ( $\eta_f = 15 \text{ Pa}\cdot\text{sec}$  at  $20^\circ\text{C}$ ) made it possible to avoid the occurrence of sedimentational instability during the experiment. In order to make use of (24), we should have complete information on the rheological properties of the matrix, i.e., the marshalite—glycerin system. The corresponding rheological curves are shown in Fig. 3. Analysis of the data by the method of linear regression made it possible to obtain a set of straight lines in logarithmic axes. The values of  $n$  corresponding to these lines are shown in logarithmic coordinates in Fig. 4 as a function of  $\varphi_1$ . Assuming that  $n(\varphi_1)$  is a power relation, we also analyzed this data by the method of linear regression and obtained the equation

$$n(\varphi_1) = 7,42\varphi_1^{4,81}. \quad (25)$$

Figure 5 shows experimental dependences of the viscosity of glass-bead—glycerin and marshalite—glycerin systems at  $S = 2.2 \text{ sec}^{-1}$  on  $\varphi_2$  and  $\varphi_1$  with  $\varphi_1 = 0$  and  $\varphi_2 = 0$ , respectively. Also shown in this figure are curves calculated from Eq. (24). Here, we assumed that  $C_F F_b/\delta_1^2 = 10^5 \text{ Pa}$ , i.e.,  $A = 3.6 \cdot 10^{-18} \text{ J}$  and  $h_b = 1 \text{ nm}$ . These conditions provided for the best agreement with the experimental data. The estimate of  $h_b$  is realistic, but the value of  $A$  is overstated compared to the values obtained for most substances —  $10^{-20}$ – $10^{-19} \text{ J}$  [18]. The deviation may be connected with the assumption of a spherical particle shape. Marshalite particles are of irregular form and have plane faces. Contacting along these faces, the particles can form stronger bonds than if the faces were differently shaped. In any case, we can take the value  $C_F F_b/\delta_1^2 = 10^5 \text{ Pa}$ , regarding it as a certain empirical characteristic of the matrix. Inserting the resulting relation  $n(\tilde{\varphi}_1)$  and the value of the parameter  $C_F F_b/\delta_1^2$  into (24), we construct the dependence of  $\eta_s/\eta_f$  on

the relative content of the finely dispersed fraction  $\varphi_1/\varphi_t$  (where  $\varphi_t = \varphi_1 + \varphi_2$ ) at  $S = 2.2 \text{ sec}^{-1}$  (see Fig. 6). It is evident from the figure that the theory agrees well with the experimental data at  $\varphi_t > 0.5$ , predicting the presence of the minimum of  $\eta_s(\varphi_1)$ . An increase in  $\varphi_t$  is accompanied by an increase in the depth of the minimum and a decrease in the corresponding value of  $\varphi_1/\varphi$ .

**Conclusion.** We determined the form of the dependence of the effective viscosity of a suspension with a power-law fluid generalized by a Newtonian fluid on particle concentration. This dependence becomes weaker as the nonlinearity of the fluid increases, while the viscosity of the suspension remains proportional to the viscosity of the fluid. The results were corrected with allowance for experimental data and numerical calculations.

We also determined the viscosity of a concentrated bidisperse system with a highly disperse structure-forming fraction for different granulometric compositions. The given theory can serve as a basis for optimizing the composition of disperse systems.

## NOTATION

Here  $A$  is the Hamaker constant;  $a$ , particle radius;  $C$ , numerical coefficient in (14);  $C_B$  and  $C_F$ , numerical coefficients used in the determination of  $\tau_B$  and  $\tau_F$ ;  $F$ , hydrodynamic resistance to the motion of the particles;  $F_b$ , force bonding the particles;  $h$ , mean distance between particle surfaces in (2);  $h_b$ , distance between the surfaces of aggregated particles;  $k$ , Boltzmann constant;  $K$  and  $n$ , parameters of the medium in (1);  $p$ , pressure;  $S$ , shear rate;  $T$ , absolute temperature;  $v$ , velocity of the fluid;  $V$ , relative velocity of the particles;  $\delta_1$  and  $\delta_2$ , diameters of particles of the fine and coarse fractions;  $\tau$ ,  $\tau_B$ , and  $\tau_F$  characteristic time of the dominant process and its value for Brownian motion and particle—particle interaction;  $\eta_s$  and  $\eta$ , viscosity of the suspension as a whole in a non-Newtonian medium (or a highly dispersed fraction);  $\eta_t$ , viscosity of the fluid;  $\eta_0$ ,  $\eta_\infty$ , values of  $\eta$  at  $S \rightarrow 0$  and  $S \rightarrow \infty$ ;  $\varphi_1$  and  $\varphi_2$  volume contents of the fine and coarse fractions;  $\varphi$ , volume fraction of particles in suspension in the non-Newtonian medium;  $\varphi_m$ , maximum dense random packing;  $\tilde{\varphi}_1$ , local value of  $\varphi_1$ ;  $\varphi_t \equiv \varphi_1 + \varphi_2$ .

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